

HÖLDER CONTINUITY OF SOLUTION MAPS TO A PARAMETRIC WEAK VECTOR EQUILIBRIUM PROBLEM

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ABSTRACT. In this paper, by using a new concept of strong convexity, we obtain sufficient conditions for Hölder continuity of the solution mapping for a parametric weak vector equilibrium problem in the case where the solution mapping is a general set-valued one. Without strong monotonicity assumptions, the Hölder continuity for solution maps to parametric weak vector optimization problems is discussed.

Keywords: vector equilibrium problem, solution mapping, Hölder continuity, linear scalarization.

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1. Introduction

Let X and Y be real topological vector spaces, and K a nonempty subset of X . Let C be a nonempty, closed, and convex cone in Y with nonempty interior, i.e., $\text{int } C \neq \emptyset$. Let $f : X \times X \rightarrow Y$ be a vector-valued bifunction. The weak vector equilibrium problem is to find $\bar{x} \in K$ such that

$$f(\bar{x}, y) \notin -\text{int } C, \quad \forall y \in K.$$

This model was extension and generalization from the equilibrium problem in [10]. It is well known that the vector equilibrium problem provides a unified model of several classes of problems, including, vector variational inequality problems, vector complementarity problems, vector optimization problems, and vector saddle point problems, so on. The existence of solutions for weak vector equilibrium problems is the most popular topics that has been studied intensively and extensively; for example, see [7, 11, 19]. By the way, when constraint set and/or objective function are perturbed by parameters, the stability analysis is also interesting topics for optimization theory. Stability analysis consider the effect of slight change on a parameter to the

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solution of problem, for examples including, lower(upper) semicontinuity, continuity, and lower(upper) Lipschitz or lower(upper) Hölder continuity. Hölder stability is one of the most studied topic in the theory of stability for vector equilibrium problems and related problems (see [1–5, 8, 9, 13–18, 20–30]). At first, the Hölder continuity of solution maps to parametric vector equilibrium problems depends upon the strong monotonicity assumptions, for example we refer the reader to [1–3, 8, 9, 22]. Such conditions lead to uniqueness of solution and inapplicable for some special cases. In many situations, the solution sets may not be a singleton. Is there a way to avoid the monotonicity assumptions. In order to answer this problem, Li et al. [16, 17] introduced Hölder related assumptions and established the Hölder continuity of solutions to generalized vector (quasi)equilibrium problems. Although, these conditions do not allow an uniqueness solution it requires to known the information of a solution set. Li and Li [18] introduced the concepts of Hölder strongly monotone and Hölder continuous with respect to the interior point of an ordering cone. By using a linear scalarization technique, they obtained the Hölder continuity of the set-valued solution sets for parametric weak vector equilibrium problems in metric spaces. Peng et al. [26] could omit the strong monotonicity assumption. Based on the linear scalarization technique, they obtained the Hölder continuity of set-valued approximate solution maps. That means the Hölder property holds for nearby original solution sets. For more related results, see [4, 25]. Recently, Anh et al., [6] gave the sufficient condition for Hölder continuity of unique solution mappings to parametric equilibrium problems, by using the strong convexity assumption. Without strong monotonicity, their results can be applicable for parametric variational inequality and optimization problems.

Based on the above literature, the aim of this paper is to establish the Hölder continuity of solution maps to a parametric weak vector equilibrium problem without the strong monotonicity assumptions and requirement for information of solution sets. Inspired by [6, 18], we introduce the new concept of strong convexity with respect to an interior point in ordering cones and establish the sufficient conditions for Hölder continuity for set-valued exact solution maps to a parametric weak vector equilibrium problem, by using a linear scalarization method. Application to a parametric weak vector optimization problem is also discussed when strong monotonicity assumptions could omit.

The structure of the paper is as follows. Section 2 presents a parametric weak generalized vector equilibrium problem and materials used in the rest of this paper. We establish, in Section 3, a sufficient condition for the Hölder continuity of the solution mapping to a parametric