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The stable equilibrium of a system of piecewise linear difference equations

Wirot Tikjha^{1,2*}, Evelina Lapierre³ and Thanin Sitthiwiratttham⁴

*Correspondence:

wirottik@psru.ac.th

¹Faculty of Science and Technology,
Pibulsongkram Rajabhat University,
Phitsanulok, Thailand

²Centre of Excellence in
Mathematics, PERDO, CHE,
Phitsanulok, Thailand

Full list of author information is
available at the end of the article

Abstract

In this article we consider the global behavior of the system of first order piecewise linear difference equations: $x_{n+1} = |x_n| - y_n + b$ and $y_{n+1} = x_n - |y_n| - d$ where the parameters b and d are any positive real numbers. We show that for any initial condition in $\mathbb{R}^2$ the solution to the system is eventually the equilibrium, $(2b + d, b)$. Moreover, the solutions of the system will reach the equilibrium within six iterations.

MSC: 39A10; 65Q10

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1 Introduction

In applications, difference equations usually describe the evolution of a certain phenomenon over the course of time. In mathematics, a difference equation produces a sequence of numbers where each term of the sequence is defined as a function of the preceding terms. For the convenience of the reader we supply the following definitions. See [1, 2]. A *system of difference equations of the first order* is a system of the form

$$\begin{cases} x_{n+1} = f(x_n, y_n), \\ y_{n+1} = g(x_n, y_n), \end{cases} \quad n = 0, 1, \dots, \quad (1)$$

where f and g are continuous functions which map $\mathbb{R}^2$ into $\mathbb{R}$.

A *solution* of the system of difference equations (1) is a sequence $\{(x_n, y_n)\}_{n=0}^{\infty}$ which satisfies the system for all $n \geq 0$. If we prescribe an *initial condition*

$$(x_0, y_0) \in \mathbb{R}^2$$

then

$$\begin{cases} x_1 = f(x_0, y_0), \\ y_1 = g(x_0, y_0), \\ \\ x_2 = f(x_1, y_1), \\ y_2 = g(x_1, y_1), \\ \\ \vdots \end{cases}$$