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A characterization of nonemptiness and boundedness of the solution set for set-valued vector equilibrium problems via scalarization and stability results

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Abstract

In this paper, the existence theorems of solutions for generalized weak vector equilibrium problems are developed in real reflexive Banach spaces. Based on recession method and scalarization technique, we derive a characterization of nonemptiness and boundedness of solution set for generalized weak vector equilibrium problems. Moreover, Painlevé–Kuratowski upper convergence of solution set is also discussed as an application, when both the objective mapping and the constraint set are perturbed by difference parameters.

Keywords: Equilibrium problem, Barrier cone, Pseudomonotone mappings, Stability analysis

Background

Let X be a real reflexive Banach space and Y be a real normed linear space. Suppose that, and $C \subseteq Y$ is a nonempty, closed and convex pointed cone with $\text{int } C \neq \emptyset$. Let $K \subseteq X$ be a non-empty subset and a set-valued function $F : K \times K \rightarrow 2^Y \setminus \{\emptyset\}$, the following generalized weak vector equilibrium problem (GWVEP) is to find $\bar{x} \in K$ such that

$$\bar{x} \in K \text{ such that } F(\bar{x}, y) \cap (-\text{int } C) = \emptyset, \forall y \in K, \quad (\text{GWVEP})$$

and the dual problem for (GWVEP), is so called (DGWVEP), is to find $\bar{x} \in K$ such that

$$\bar{x} \in K \text{ such that } F(y, \bar{x}) \cap (\text{int } C) = \emptyset, \forall y \in K. \quad (\text{DGWVEP})$$

Both (GWVEP) and (DGWVEP) have been extensively studied by many authors (see Ansari and Flores-Bazán 2006; Ansari et al. 2001a, b, 2002; Flores-Bazán and Flores-Bazán 2003; Ansari et al. 2001; Sadeqi and Alizadeh 2011; Zhong et al. 2011). An important and interesting topic for (GWVEP) and (DGWVEP) is to study the nonemptiness and boundedness of the solution sets. As far as we known, the first paper which discussed this issues was Flores-Bazán and Flores-Bazán (2003) in the case where F is vector-valued. They studied the existence of solutions of (GWVEP) under the asymptotic analysis, where neither compactness of K nor any coercivity condition is assumed in