

# Weakly Prime and Weakly Semiprime Left ideal in Ternary Semigroups

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**ABSTRACT**— *The purpose of this paper is to introduce the notion of a weakly prime ideals in ternary semigroups, we study left ideals, weakly prime and weakly semiprime ideals in ternary semigroups. Some characterizations of  $\Gamma$ -weakly semiprime and weakly prime ideals are obtained.*

**Keywords**— ternary semigroup, weakly semiprime, weakly prime, m-Systems, n-Systems

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## 1. INTRODUCTION

There is a large literature dealing with ternary operations. The notion of ternary semigroup is a natural generalization of ternary group. Algebraic structures play a prominent role in mathematics with wide applications in many disciplines such as computer sciences, information sciences, engineering, physics etc. Banach find some applications in ternary semigroup. He gave an example to show that a ternary semigroup is not necessarily reduce to an ordinary semigroup. Los [9] studied some properties of ternary semigroup and proved that every ternary semigroup can be embedded in a semigroup.

In 1932, Lehmer introduced the concept of ternary semigroup [8]. The algebraic structures of ternary semigroups were also studied by some authors, for example, Sioson studied ideals in ternary semigroups [17]. A non-empty set  $S$  is called a ternary semigroup if there exists a ternary operation  $[\ ]: S \times S \times S \rightarrow S$ ; satisfying the following identity, for any  $x_1, x_2, x_3, x_4, x_5 \in S$ ,

$$[[x_1x_2x_3]x_4x_5] = [x_1[x_2x_3x_4]x_5] = [x_1x_2[x_3x_4x_5]]$$

Any semigroup can be reduced to a ternary semigroup. However, Banach showed that a ternary semigroup does not necessarily reduce to a semigroup by this example. Let  $S = \{-i, 0-i\}$ , where  $S$  is a ternary semigroup while  $S$  is not a semigroup under the multiplication over complex numbers. Los showed that every ternary semigroup can be embedded in a semigroup [9]. In this paper we characterize the ternary semigroups, we study left ideals, weakly prime and weakly semiprime ideals in ternary semigroups.

## 2. BASIC PROPERTIES

In this section we refer to [2, 3] for some elementary aspects and quote few definitions and examples which are essential to step up this study. For more detail we refer to the papers in the references.

**Definition 2.1.** [2, 3] A nonempty set  $S$  with a ternary operation  $(x, y, z) \mapsto [xyz]$  satisfying the associative law, that is, for any  $x_1, x_2, x_3, x_4, x_5 \in S$ ,

$$[[x_1x_2x_3]x_4x_5] = [x_1[x_2x_3x_4]x_5] = [x_1x_2[x_3x_4x_5]]$$

is called a ternary semigroup.