

On Classical Quasi-Primary Radical of Submodules and Classical Quasi-Primary Radical Formula of Submodules

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Received: 15 October 2014, Revised: 3 March 2015, Accepted: 5 April 2015

Abstract

In this paper we characterize the classical quasi-primary radical of submodules and classical quasi-primary radical formula of modules over commutative rings with identity. These are extended from radical, radical primary, and radical formula of submodules, respectively. Finally, we obtain necessary and sufficient conditions of a submodule in order to be a top classical quasi-primary radical formula of submodules.

Keywords: Classical quasi-primary submodule, classical quasi-primary radical of submodule, classical quasi-primary radical formula of submodule

Introduction

Throughout this paper all rings are commutative with identity and all modules are unitary. We recall that a proper ideal P of R is called a *primary ideal* if $ab \in P$, where $a, b \in R$, implies that either $a \in P$ or $b^n \in P$, for some positive integer n [1]. The notion of primary ideal was generalized by Fuchs [1] through defining an ideal P of a ring R to be *quasi primary* if its radical is a prime ideal, i.e., if $ab \in P$, where $a, b \in R$, then either $a^n \in P$ or $b^n \in P$, for some positive integer n (see also [2,3]). A subset N of the R -module M is a submodule, expressed $N \leq M$, if it is a subgroup that is closed under the R -action: For all $r \in R$ and $n \in N$, we have $rn \in N$. We say that a submodule N of M is proper if $N \neq M$. A proper submodule N of an R -module M is a *primary submodule* of M if for $m \in M$ and $r \in R$ such that $rm \in N$, then $m \in N$ or $r \in \sqrt{(N : M)} = \{a \in R \mid a^n M \subseteq N, \text{ for some positive integer } n\}$. An R -module M is a *primary module* if every proper submodule N of M is a primary submodule of M (see, for example, [4-8]). A *classical primary submodule* in M as a proper submodule N of M such that if $abK \subseteq N$, where $a, b \in R$ and $K \leq M$, then either $aK \subseteq N$ or $b^n K \subseteq N$ for some positive integer n . Clearly, in case $M = R$, where R is any commutative ring, classical primary submodules coincide with primary ideals [9-11]. A *classical quasi-primary submodule* in M as a proper submodule N of M such that if $abK \subseteq N$, where $a, b \in R$ and $K \leq M$, then either $a^n K \subseteq N$ or $b^n K \subseteq N$ for some positive integer n . The idea of decomposition of submodules into classical primary submodules was introduced by Bazar and Behboodi in [9]. The *primary radical* of N in M , denoted by $\text{prad}_M(N)$, is defined to