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Existence and continuity for the ε -approximation equilibrium problems in Hadamard spaces

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Abstract

In this paper, the existence of ε -approximate equilibrium points for a bifunction is proved under suitable conditions in the framework of a Hadamard space. We also give the sufficient conditions for the continuity of ε -approximate solution maps to equilibrium problems. Then we apply our results to constrained minimization problems and Nash-equilibrium problems.

MSC: Primary 47J25; 47N10; 34G20; 65J15

Keywords: ε -approximation solution; continuity; equilibrium problem; Hadamard space

1 Introduction

Let K be a nonempty subset of a linear space X and $f : K \times K \rightarrow \mathbb{R}$ a bifunction. The equilibrium problem, for short (EP), is defined as follows:

$$\text{Find } x \in K \text{ such that } f(x, y) \geq 0, \quad \forall y \in K. \quad (1.1)$$

(EP) was formulated by Blum and Oettli [1] in 1994. It is shown in the paper that (EP) includes many classical problems in optimization, such as variational inequality, constrained minimization, fixed point, Nash-equilibrium problems, *etc.* Since that time, there has been extensive study in various directions; see [2–4] and references therein. Almost all extended and generalized results are studied in linear spaces. Actually, some applications in optimization problems cannot be considered in linear space. Recently, in the past decade, many optimization problems were investigated on nonlinear space, such as Riemannian manifolds, hyperbolic space and geodesic spaces, for example in [5–7]. In 2012, Colao *et al.* [8] first established the existence for an equilibrium point in the setting of Hadamard manifolds. By using an analogous KKM lemma, the authors developed an equilibrium theory and gave applications to optimization-related problems in Hadamard manifolds. This paper is an inspiration for studying the generalized and/or application of (EP) in Hadamard manifolds; see [5, 9, 10]. The KKM lemma is the main tool for proving an existence result for (EP). The proof of the KKM lemma relies on the fact that the closed convex hull of any finite set is compact. It was mentioned by Kopecká and Reich [11] that, in any Hadamard