



On Left Primary and Weakly Left Primary Ideals in Γ -LA-Rings

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ABSTRACT

In this paper, we study left ideals, left primary and weakly left primary ideals in Γ -LA-rings. Some characterizations of left primary and weakly left primary ideals are obtained. Moreover, we investigate relationships left primary and weakly left primary ideals in Γ -LA-rings. Finally, we obtain necessary and sufficient conditions of a weakly left primary ideal to be a left primary ideals in Γ -LA-rings.

Keywords: Γ -LA-ring, left primary ideal, weakly left primary ideal, left ideal.

1. INTRODUCTION

Abel-Grassmann's groupoid (AG-groupoid) is the generalization of semigroup theory with the wide range of usages in theory of flocks [6]. The fundamentals of this non-associative algebraic structure were the first discovered by Kazim and Naseeruddin [1]. AG-groupoid is a non-associative algebraic structure mid way between a groupoid and a commutative semigroup. It is interesting to note that an AG-groupoid with right identity becomes

a commutative monoid [4]. This structure is closely related with a commutative semigroup because if an AG-groupoid contains a right identity, then it becomes a commutative monoid [4]. A left identity in an AG-groupoid is unique. Ideals in AG-groupoids have been discussed by Mushtaq and Yousuf [4, 5].

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In 1981, the notion of Γ -semigroups was introduced by Sen. Let S and Γ be any nonempty sets. If there exists a mapping $S \times \Gamma \times S \rightarrow S$ written (a, α, c) by $a\alpha c$, S is called a Γ -semigroup if S satisfies the identity:

$$(a\alpha b)\beta c = a\alpha(b\beta c)$$

for all $a, b, c \in S$ and $\alpha, \beta \in \Gamma$. A Γ -AG-groupoids analogous to Γ -semigroups. A groupoid S is called a Γ -AG-groupoid if it satisfies the left invertive law:

$$(a\gamma b)\delta c = (c\gamma b)\delta a$$

for all $a, b, c, d \in S$ and $\gamma, \delta \in \Gamma$ [2]. This structure is also known as left almost semigroup (LA-semigroup).

S.M. Yusuf in [18] introduces the concept of a left almost ring (LA-ring). That is, a non-empty set R with two binary operations "+" and "." is called a left almost ring, if $(R, +)$ is a LA-group, $(R, \cdot)$ is a LA-semigroup and distributive laws of "." over "+" holds. Further in [12] T. Shah and I. Rehman generalize the notions of commutative semigroup rings into LA-semigroup LA-rings. However T. Shah and Fazal ur Rehman in [12] generalize the notion of a LA-ring into an nLA-ring. A near left almost ring (nLA-ring) N is a LAgroup under "+", a LA-semigroup under "." and left distributive property of "." over "+" holds.

T. Shah, Fazal ur Rehman and M. Raees asserted that a commutative ring $(R, +, \cdot)$, we can always obtain a LA-ring $(R, \oplus, \cdot)$ by defining, for $a, b, c \in R$, $a \oplus b = b - a$ and $a \cdot b$ is same as in the ring. Furthermore, in this paper we characterize the left primary and weakly left primary ideals in Γ -LA-rings. Moreover, we investigate relationships left primary and weakly left primary ideals in Γ -LA-rings. Finally, we obtain necessary and sufficient conditions of a weakly left primary ideal to be a left primary ideals in Γ -LARings.

2. IDEALS IN Γ -LA-RINGS

The results of the following lemmas seem play an important role to study Γ -LA-ring; these facts will be used so frequently that normally we shall make no reference to this lemma.

Definition 2.1. Let $(R, +)$ and $(\Gamma, +)$ be two LA-groups, R is called a Γ -left almost ring (Γ -LA-ring) if there exists a mapping $R \times \Gamma \times R \rightarrow R$ by $(a, \alpha, b) \mapsto a\alpha b$, for all $a, b \in R$ and $\alpha \in \Gamma$ satisfying the following conditions

1. $a\alpha(b+c) = a\alpha b + a\alpha c$
2. $(a+b)\alpha c = a\alpha c + b\alpha c$
3. $a(\alpha+\beta)b = a\alpha b + a\beta b$
4. $(a\alpha b)\beta c = (c\alpha b)\beta a$, for all $a, b, c \in R$ and $\alpha, \beta \in \Gamma$.

Lemma 2.2. If R is a Γ -LA-ring with left identity, then $a\gamma b = a\beta b$, for all $a, b \in R$ and $\gamma, \beta \in \Gamma$.

Proof. Let R is a Γ -LA-ring and e be the left identity of $a, b \in R$ and let $\gamma, \beta \in \Gamma$ therefore we have

$$\begin{aligned} a\gamma b &= a\gamma(e\beta b) \\ &= e\gamma(a\beta b) \\ &= a\beta b. \end{aligned}$$

Hence $a\gamma b = a\beta b$.

Lemma 2.3. Let R be a Γ -LA-ring with left identity e . Then $R\Gamma R = R$ and $R = e\Gamma R = R\Gamma e$.

Proof. Let R be a Γ -LA-ring with left identity e and let $r \in R$ then $r = e\alpha r \in R\Gamma R$, for all $\alpha \in \Gamma$, so that $R \subseteq R\Gamma R$. Since R is a Γ -LA-ring, we have $R\Gamma R \subseteq R$. Thus $R\Gamma R = R$. Now as e is a left identity in R , $e\alpha a = a$, for all $a \in R$ and $\alpha \in \Gamma$. Then $R = e\Gamma R$. Since $(a\alpha b)\beta c = (c\alpha b)\beta a$, for all $a, b, c \in R$ and $\alpha, \beta \in \Gamma$, we have $(R\Gamma R)\Gamma e = (e\Gamma R)\Gamma R$. Now,

$$R\Gamma e = (R\Gamma R)\Gamma e = (e\Gamma R)\Gamma R = R\Gamma R = R.$$

Hence $R = e\Gamma R = R\Gamma e$.

Definition 2.4. A nonempty subset I of a Γ -LA-ring R is a subring of R if under the binary operations in R , form a Γ -LA-ring.

Definition 2.5. A subring I of R is called a left (right) ideal of R if $R\Gamma I \subseteq I$ ($I\Gamma R \subseteq I$) and is called ideal if it is left as well as right ideal.

Lemma 2.6. If R is a Γ -LA-ring with left identity, then every right ideal is a left ideal.

Proof. Let R be a Γ -LA-ring with left identity and let A be a right ideal of R . Then for $a \in A$, $r \in R$ and $\alpha \in \Gamma$, consider

$$\begin{aligned} r\alpha a &= (e\beta r)\alpha a \\ &= (a\beta r)\alpha e \\ &\in (A\Gamma R)\Gamma R \\ &\subseteq A\Gamma R \\ &\subseteq A, \end{aligned}$$

where e is a left identity and $\beta \in \Gamma$, that is $r\alpha a \in A$. Therefore A is left ideal of R .

Lemma 2.7. If I is a left ideal of a Γ -LA-ring R with left identity, and if for any $a \in R$, $\gamma \in \Gamma$, then $a\gamma I$ is a left ideal of R .

Proof. Let I be a left ideal of R , consider

$$\begin{aligned} s\gamma(a\gamma i) &= (e\gamma s)\gamma(a\gamma i) \\ &= (e\gamma a)\gamma(s\gamma i) \\ &= a\gamma(s\gamma i) \in a\gamma I \end{aligned}$$

and $(a\gamma i) + (a\gamma j) = a\gamma(i + j) \in a\gamma I$. Hence $a\gamma I$ is a left ideal of R .

Lemma 2.8. Let R be a Γ -LA-ring with left identity, and $a \in R$, $\gamma \in \Gamma$. Then $R\gamma a$ is a left ideal of R .

Proof. Let R be a Γ -LA-ring with left identity, and $a \in R$, $\gamma \in \Gamma$. Then