

Some Basic Properties of Almost-Prime Left Ideals in Γ -Semirings

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ABSTRACT

The purpose of this paper was to introduce the notion of almost-prime left ideals in Γ -semirings by studying prime and weakly almost-prime left ideals in Γ -semirings. Some characterizations of almost-prime and weakly almost-prime left ideals were obtained. Moreover, the relationships between almost-prime and weakly almost-prime left ideals in Γ -semirings were investigated.

Keywords: Γ -semiring, ideal, quasi-ideal, almost-prime left ideal, weakly almost-prime left ideal

INTRODUCTION

The notion of Γ -semirings, was introduced in 1995 (Murali Krishna Rao, 1995) that is, a semigroup S is called a Γ -semiring if Γ is a semigroup and there exists a mapping

$$S \times \Gamma \times S \rightarrow S$$

satisfying some certain properties. (Dutta and Sardar, 2002a), gave the meaning of left and right operator semirings for a given Γ -semiring. Moreover, some relationships between Γ -semirings and their left and right operator semirings have been illustrated (Dutta and Sardar, 2002b).

It is natural to extend the concept of quasi-ideals in Γ -semirings and this was done by Chinram (2008) as a generalization of quasi-ideals in Γ -semigroups. The Γ -semirings introduced by Dutta and Sardar (2002c) and Chinram (2008) are different.

In Γ -semirings, the properties of their ideals, prime ideals, semiprime ideals and their generalizations play an important role in the theory of their structure; however, the properties of an ideal in semirings and Γ -semirings are somewhat different from the properties of the usual ring

ideals (Dutta and Sardar, 2000).

BASIC RESULTS

This section refers to Sardar and Dasgupta (2004), Jagatap and Pawar (2009), and Sardar and Mukhopadhyay (2010) for some elementary aspects and quotes a few definitions and examples which are essential to step through this study. More detail can be found in the papers in the references.

Definition 1. (Sardar and Mukhopadhyay, 2010)

Let $(S, +)$ and $(\Gamma, +)$ be commutative semigroups. Then, we call S a Γ -semiring if there exists a map $S \times \Gamma \times S \rightarrow S$ written (x, γ, y) by $x\gamma y$, such that it satisfies the following axioms (Equations 1–3) for all $x, y, z \in S$ and $\gamma, \alpha \in \Gamma$:

$$x\gamma(y+z) = x\gamma y + x\gamma z \text{ and } (x+y)\gamma z = x\gamma z + y\gamma z \quad (1)$$

$$x(\gamma+\alpha)y = x\gamma y + x\alpha y \quad (2)$$

$$(x\gamma y)\alpha z = x\gamma(y\alpha z) \quad (3)$$

Clearly, every semiring S is a Γ -semiring but not conversely. For this, let us consider the following example.