

Research Article

A Note on Continuity of Solution Set for Parametric Weak Vector Equilibrium Problems

Pakkapon Preechasilp¹ and Rabian Wangkeeree²

¹Program in Mathematics, Faculty of Education, Pibulsongkram Rajabhat University, Phitsanulok 65000, Thailand

²Department of Mathematics, Faculty of Science, Naresuan University, Phitsanulok 65000, Thailand

Correspondence should be addressed to Pakkapon Preechasilp; preechasilpp@gmail.com

Received 31 August 2014; Revised 22 December 2014; Accepted 11 January 2015

Academic Editor: Yunhai Xiao

Copyright © 2015 P. Preechasilp and R. Wangkeeree. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

We consider the parametric weak vector equilibrium problem. By using a weaker assumption of Peng and Chang (2014), the sufficient conditions for continuity of the solution mappings to a parametric weak vector equilibrium problem are established. Examples are provided to illustrate the essentialness of imposed assumptions. As advantages of the results, we derive the continuity of solution mappings for vector optimization problems.

1. Introduction

It is well known that the vector equilibrium problem provides a unified model of several classes of problems, including vector variational inequality problems, vector complementarity problems, vector optimization problems, and vector saddle point problems. There are many papers which have discussed the existence results for different types of vector equilibrium problems (see [1–3] and references therein).

In 2008 Gong [4] studied parametric vector equilibrium problems. Based on a scalarization representation of the solution mapping and the property involving the union of a family of lower semicontinuous set-valued mappings of Cheng and Zhu [5], they established the sufficient conditions for the continuity of the solution set mapping for the mixed parametric monotone weak vector equilibrium problems in topological vector spaces. In the same year, Gong and Yao [6] discussed the lower semicontinuity of the efficient solution mappings to a parametric strong vector equilibrium problem with C -strict monotonicity of a vector-valued function, by using a scalarization method and density result. In 2009, Xu and Li [7] presented a new proof of lower semicontinuity of the set of efficient solutions to a parametric strong vector equilibrium problem, which is different from the one used in [6]. In 2010, Chen and Li [8] discussed and improved

the lower semicontinuity and continuity results of efficient solution mappings to a parametric strong vector equilibrium problem in [4, 6], without the uniform compactness assumption. By virtue of the scalarization technique, [4, 6–8] have discussed the lower semicontinuity, in the case that ξ -efficient solution set is a singleton. However, in practical, the ξ -solution set may not be singleton but a general set. Recently, by using a weak assumption, Peng and Chang [9] discussed the lower semicontinuity of solution maps for parametric weak vector equilibrium problem under the case that the ξ -efficient solution mapping may not be single-valued as follows. Unfortunately, the results obtained in the corresponding papers [4, 6–9] cannot be used in the case of vector optimization problems. Hence, in this paper, we study the lower semicontinuity of the set of efficient solutions for parametric weak vector equilibrium problems when the ξ -efficient solution set is a general set. Moreover, our theorems can apply for vector optimization problems.

The structure of the paper is as follows. Section 2 presents the efficient solutions to parametric weak vector equilibrium problems and materials used in the rest of this paper. We establish, in Section 3, a sufficient condition for the continuity of the efficient solution mappings. We give some examples to illustrate that our main results are different from the corresponding ones in the literature. Section 4 is reserved