

On the Classical Prime Radical Formula and Classical Prime of Semimodules

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ABSTRACT— Let R be a commutative semiring and M an R semimodule. A proper subsemimodule N of M is called a classical prime subsemimodule, if for any $a, b \in R$ and $m \in M$, $abm \in N$ implies that $am \in N$ or $bm \in N$. We will introduce and study the notion of prime bases for classical prime subsemimodules and utilize them to derive some formulas on the classical prime radical of subsemimodules of a semimodule. In particular, we study some basic properties of prime radical and classical prime radical of subsemimodule in M . Moreover, we investigate relationships between classical prime radical and prime radical of subsemimodule in M .

Keywords— prime subsemimodule, classical prime subsemimodule, prime radical, classical prime radical, prime ideal

1. INTRODUCTION

Throughout this paper a semiring will be defined as follows: A semiring is a set R together with two binary operations called addition "+" and multiplication "·" such that $(R, +)$ is a commutative semigroup and $(R, \cdot)$ is semigroup; connecting the two algebraic structures are the distributive laws : $a(b+c) = ab+ac$ and $(a+b)c = ac+bc$ for all $a, b, c \in R$. A subset A of a semiring R is called an ideal of R if for $a, b \in A$ and $r \in R$, $a+b \in A$, $ar \in A$ and $ra \in A$. A proper ideal P of R is called a prime ideal if $ab \in P$, where $a, b \in R$, implies that either $a \in P$ or $b \in P$. A proper ideal P of R is said to be quasi-primary if for all $a, b \in R$, $ab \in P$ implies that either $a^n \in P$ or $b^n \in P$, for some positive integer n . Clearly every prime is a quasi-primary. A semimodule M over a semiring R is a commutative monoid M , together with a function $R \times M \rightarrow M$, defined by $(r, m) \mapsto rm$ such that:

1. $r(m+n) = rm + rn$
2. $(r+s)m = rm + sm$
3. $(rs)m = r(sm)$
4. $1m = m$

for all $m, n \in M$ and $r, s \in R$. Clearly every ring is a semiring and hence every module over a ring R is a left semimodule over a semiring R . A nonempty subset N of a R -semimodule M is called subsemimodule of M if N is closed under addition and closed under scalar multiplication. A proper subsemimodule N of an R -semimodule M is said to be prime if $rm \in N$, $r \in R, m \in M$, then either $m \in N$ or $rM \subseteq N$.

J. Saffar Ardabili, S. Motmaen and A. Yousefian Darani in (2011) defined a different class of subsemimodules and called it classical prime. A proper subsemimodule N of M is said to be classical prime when for $a, b \in R$ and $m \in M$, $abm \in N$ implies that $am \in N$ or $bm \in N$. A classical prime radical of N in M , denoted by

Weakly Prime and Weakly Semiprime Left ideal in Ternary Semigroups

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ABSTRACT— *The purpose of this paper is to introduce the notion of a weakly prime ideals in ternary semigroups, we study left ideals, weakly prime and weakly semiprime ideals in ternary semigroups. Some characterizations of Γ -weakly semiprime and weakly prime ideals are obtained.*

Keywords— ternary semigroup, weakly semiprime, weakly prime, m-Systems, n-Systems

1. INTRODUCTION

There is a large literature dealing with ternary operations. The notion of ternary semigroup is a natural generalization of ternary group. Algebraic structures play a prominent role in mathematics with wide applications in many disciplines such as computer sciences, information sciences, engineering, physics etc. Banach find some applications in ternary semigroup. He gave an example to show that a ternary semigroup is not necessarily reduce to an ordinary semigroup. Los [9] studied some properties of ternary semigroup and proved that every ternary semigroup can be embedded in a semigroup.

In 1932, Lehmer introduced the concept of ternary semigroup [8]. The algebraic structures of ternary semigroups were also studied by some authors, for example, Sioson studied ideals in ternary semigroups [17]. A non-empty set S is called a ternary semigroup if there exists a ternary operation $[\]: S \times S \times S \rightarrow S$; satisfying the following identity, for any $x_1, x_2, x_3, x_4, x_5 \in S$,

$$[[x_1 x_2 x_3] x_4 x_5] = [x_1 [x_2 x_3 x_4] x_5] = [x_1 x_2 [x_3 x_4 x_5]]$$

Any semigroup can be reduced to a ternary semigroup. However, Banach showed that a ternary semigroup does not necessarily reduce to a semigroup by this example. Let $S = \{-i, 0 - i\}$, where S is a ternary semigroup while S is not a semigroup under the multiplication over complex numbers. Los showed that every ternary semigroup can be embedded in a semigroup [9]. In this paper we characterize the ternary semigroups, we study left ideals, weakly prime and weakly semiprime ideals in ternary semigroups.

2. BASIC PROPERTIES

In this section we refer to [2, 3] for some elementary aspects and quote few definitions and examples which are essential to step up this study. For more detail we refer to the papers in the references.

Definition 2.1. [2, 3] A nonempty set S with a ternary operation $(x, y, z) \mapsto [xyz]$ satisfying the associative law, that is, for any $x_1, x_2, x_3, x_4, x_5 \in S$,

$$[[x_1 x_2 x_3] x_4 x_5] = [x_1 [x_2 x_3 x_4] x_5] = [x_1 x_2 [x_3 x_4 x_5]]$$

is called a ternary semigroup.