



Some Basic Properties of medial Γ -Q-Algebras

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Abstract

The purpose of this paper is to introduce the notion of Γ -Q-algebras, we study medial Γ -Q-algebras.

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INTRODUCTION

A Q-algebra is a nonempty set X with a constant 0 and a binary operation “ $*$ ” satisfying axioms: for each $x, y, z \in X$,

1. $x * x = 0$,
2. $x * 0 = x$,
3. $(x * y) * z = (x * z) * y$

Y. Imai and K. Iseki introduced two classes of abstract algebras: BCK-algebras and BCI-algebras ([5, 6]). It is known that the class of BCK-algebras is a proper subclass of the class of BCI-algebras. In [3, 5], Q. P. Hu and X. Li introduced a wide class of abstracts: BCH-algebras. They have shown that the class of BCI-algebras is a proper subclass of the class of BCH-algebras. In [8], J. Neggers, etc introduced the notion of an Q-algebra as a dualization of a generation of a BCK/BCI-algebras. In [1], Ahn and Kim introduced the notion of QS-algebra which is a generalization of Q-algebras. It is easy to see that every Q-algebras is Γ -Q-algebras.

Our aim in this paper is to introduce the notion of a Γ -Q-algebras, we study medial Γ -Q-algebras.

BASIC RESULTS

In this section is to introduce the notion of a Γ -Q-algebras.

Definition 2.1. Let X and Γ be any nonempty sets. The structure $(\Gamma, X; 0)$ is called a Γ -Q-algebra if there exists a mapping $X \times \Gamma \times X \rightarrow X$ written as (x, γ, y) by $x\gamma y$, that satisfies the following condition

1. $x\gamma x = 0$, for all $x \in X$ and $\gamma \in \Gamma$,
2. $x\gamma 0 = x$, for all $x \in X$ and $\gamma \in \Gamma$,
3. $(x\gamma y)\beta z = (x\gamma z)\beta y$ for all $x, y, z \in X$ and $\gamma, \alpha \in \Gamma$.

Throughout this paper X will denote a Γ -Q-algebra. We introduce a relation $\leq$ on X by $x \leq y$ if and only if $x\gamma y = 0$.

Example 2.2. Let $(\Gamma, X, 0)$ be an arbitrary CI-algebra and Γ any nonempty set. Define a mapping $X \times \Gamma \times X \rightarrow X$, by $x\gamma y \mapsto x * y$ for all $x, y \in X$ and $\gamma \in \Gamma$. It is easy to see that X is a Γ -Q-algebra. Indeed,

$$\begin{aligned} 1. \quad x\gamma x &= x * x \\ &= 0, \\ 2. \quad x\gamma 0 &= x * 0 \\ &= x, \\ 3. \quad (x\gamma y)\beta z &= (x * y)\beta z \\ &= (x * y) * z \\ &= (x * z) * y \\ &= (x\gamma z)\beta y \end{aligned}$$

$x, y, z \in X$ and $\gamma, \alpha \in \Gamma$. Thus every Q-algebra implies a Γ -Q-algebra.

Example 2.3. Let $X = \{0, a, b, c, d\}$ in which “ $\cdot$ ” is defined by

$\cdot$	0	a	b	c	d
0	0	a	b	c	d
a	0	0	b	b	d
b	0	a	0	a	d
c	0	0	0	0	d
d	d	d	d	d	0

Let $\Gamma \neq \emptyset$. Define a mapping $X \times \Gamma \times X \rightarrow X$ by $x\gamma y = yx$ for all $x, y \in X$ and $\gamma \in \Gamma$. Then X is a Γ -Q-algebra. But it is not a Q-algebra because $d \cdot 0 = d \neq 0$.

Lemma 2.4. Let X be a Γ -Q-algebra. Then $x\gamma y = x\beta y$ for any $x, y \in X$ and $\gamma, \beta \in \Gamma$.

Proof. Let $x, y \in X$ and $\gamma, \beta \in \Gamma$. Then

$$\begin{aligned} x\gamma y &= (x\beta 0)\gamma y \\ &= (x\beta y)\gamma 0 \\ &= x\beta y. \end{aligned}$$

Hence $x\gamma y = x\beta y$. □

Theorem 2.5. If X is a Γ -Q-algebra, then $(x\gamma(x\beta y))\alpha y = 0$, for any $x, y \in X$ and $\gamma, \alpha, \beta \in \Gamma$.

Proof. Let $x, y \in X$ and $\gamma, \alpha, \beta \in \Gamma$. Then

$$\begin{aligned} (x\gamma(x\beta y))\alpha y &= (x\gamma y)\alpha(x\beta y) \\ &= (x\gamma y)\alpha(x\gamma y) \\ &= 0. \end{aligned}$$



Hence $(x\gamma(x\beta y))\alpha y = 0$. □

Theorem 2.6. If X is a Γ -Q-algebra, then $0\gamma(x\alpha y) = (0\gamma x)\alpha(0\gamma y)$ for any $x, y \in X$ and $\gamma, \alpha \in \Gamma$.

Proof. Let $x, y \in X$ and $\gamma, \alpha, \beta \in \Gamma$. Then

$$\begin{aligned} 0\gamma(x\alpha y) &= ((0\beta y)\delta(0\beta y))\gamma(x\alpha y) \\ &= ((0\beta y)\delta(x\alpha y))\gamma(0\beta y) \\ &= (((x\beta y)\gamma x)\delta(x\alpha y))\gamma(0\beta y) \\ &= (((x\beta y)\gamma(x\alpha y))\delta x)\gamma(0\beta y) \\ &= (0\delta x)\gamma(0\beta y) \\ &= (0\gamma x)\alpha(0\gamma y). \end{aligned}$$

Hence $0\gamma(x\alpha y) = (0\gamma x)\alpha(0\gamma y)$. □

MEDIAL Γ -Q-ALGEBRAS

The results of the following lemmas seem play an important role to study a medial Γ -Q-algebra; these facts will be used so frequently that normally we shall make no reference to this definition.

Definition 3.1. A Γ -Q-algebra X is said to be medial if it satisfies the following property:

$$(x\gamma y)\alpha(z\beta u) = (x\gamma z)\alpha(y\beta u),$$

for any $x, y, z, u \in X$ and $\gamma, \alpha, \beta \in \Gamma$.

Lemma 3.2. If X is a medial Γ -Q-algebra, then $y\gamma x = 0\alpha(x\gamma y)$ for any $x, y \in X$ and $\gamma, \alpha \in \Gamma$.

Proof. Let $x, y \in X$ and $\gamma, \alpha, \beta \in \Gamma$. Then

$$\begin{aligned} y\gamma x &= (y\gamma x)\alpha 0 \\ &= (y\gamma x)\alpha(y\gamma y) \\ &= (y\gamma y)\alpha(x\gamma y) \\ &= 0\alpha(x\gamma y). \end{aligned}$$

Hence $y\gamma x = 0\alpha(x\gamma y)$. □

Lemma 3.3. If X is a medial Γ -Q-algebra, then $x\gamma(y\alpha z) = z\gamma(y\alpha x)$ for any $x, y, z \in X$ and $\gamma, \alpha \in \Gamma$.

Proof. Let $x, y, z \in X$ and $\gamma, \alpha, \beta \in \Gamma$. Then

$$\begin{aligned} x\gamma(y\alpha z) &= 0\beta((y\alpha z)\gamma x) \\ &= 0\beta((y\alpha x)\gamma z) \\ &= z\gamma(y\alpha x). \end{aligned}$$

Hence $x\gamma(y\alpha z) = z\gamma(y\alpha x)$. □

Lemma 3.4. If X is a medial Γ -Q-algebra, then $x\gamma(x\alpha y) = y$ for any $x, y \in X$ and $\gamma, \alpha \in \Gamma$.

Proof. Let $x, y \in X$ and $\gamma, \alpha, \beta \in \Gamma$. Then

$$\begin{aligned}
 x\gamma(x\alpha y) &= 0\beta((x\alpha y)\gamma x) \\
 &= 0\beta((x\alpha x)\gamma y) \\
 &= y\gamma(x\alpha x) \\
 &= y\gamma 0 \\
 &= y\gamma 0.
 \end{aligned}$$

Hence $x\gamma(x\alpha y) = y$. □

Lemma 3.5. If X is a medial Γ -Q-algebra, then $0\gamma(0\alpha y) = y$ for any $y \in X$ and $\gamma, \alpha \in \Gamma$.

Proof. Let $y \in X$ and $\gamma, \alpha \in \Gamma$. Then

$$\begin{aligned}
 0\gamma(0\alpha y) &= y\alpha 0 \\
 &= y.
 \end{aligned}$$

Hence $0\gamma(0\alpha y) = y$. □

Theorem 3.6. A Γ -Q-algebra X is medial if and only if it satisfies one of the following conditions: for any $x, y, z \in X$ and $\gamma, \alpha, \beta \in \Gamma$.

1. $y\gamma x = 0\alpha(x\gamma y)$
2. $x\gamma(y\alpha z) = z\gamma(y\alpha x)$
3. $x\gamma(x\alpha y) = y$
4. $0\gamma(0\alpha y) = y$.

Proof: $\Rightarrow$ It is clear.

$\Leftarrow$. Let $x, y, z \in X$ and $\gamma, \alpha, \beta \in \Gamma$. Then

$$\begin{aligned}
 (x\gamma y)\alpha(z\beta u) &= u\alpha(z\beta(x\gamma y)) \\
 &= u\alpha(y\beta(x\gamma z)) \\
 &= (xyz)\alpha(y\beta u)
 \end{aligned}$$

Hence $(x\gamma y)\alpha(z\beta u) = (xyz)\alpha(y\beta u)$, which completes the proof. □

CONCLUSIONS

Many new classes of Γ -Q-algebras have been discovered recently. All these have attracted researchers of the field to investigate these newly discovered classes in detail. This article investigates we study medial Γ -Q-algebras.

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