

DEWMA Chart with Transformed Exponential Observations

W. Suriyakat

*Mathematics and Statistics, Faculty of Science and Technology,
Pibulsongkram Rajabhat University, Phitsanulok, 65000, Thailand*

Abstract

Control chart is widely used in many applications for monitoring or detecting the process. The double exponentially weighted moving average (DEWMA) chart is used for detecting shifts in the process mean. In this paper, we study the average run length (ARL) properties of one-sided DEWMA chart with transformed exponential observations. Using simulation study the performance of the chart. Subsequently, the performance of the DEWMA chart with transformed exponential observations is compared with the exponentially weighted moving average (EWMA) chart. It is shown that the DEWMA chart is more efficient when compared with EWMA chart for detecting mean shift.

Keywords: ARL; Control Chart; Double Exponentially Weighted Moving Average; Transformation

1. Introduction

Control chart is important tool of Statistical Process Control (SPC) for monitoring in the process, and was used in many applications i.e. finance [1], computer science [2] and health care [3]. The double exponentially weighted moving average (DEWMA) chart is a method for detecting shifts in the process mean, and was introduced by [4]. A DEWMA chart with variable sampling intervals (VSI-DEWMA) for detecting shifts in the process mean by [5] and [6] proposed a DEWMA chart for monitoring Poisson data, using simulation and compared average run lengths and standard deviations of run lengths of the Poisson DEWMA chart with those of the Poisson EWMA chart. The EWMA technique was extended by [7] to DEWMA technique and shown that DEWMA mean chart perform better than EWMA mean chart in detecting small mean shifts ranging from 0.1σ to 0.5σ , and that the two types of charts perform similarly when mean shifts are larger than 0.5σ . The DEWMA chart for monitoring the process mean based on zero-state and worst-case average run length (ARL) measures were studied by [8]. Recently, [9] investigated the sum of squares DEWMA (SS-DEWMA) chart. A transformed exponential observation is usually studied in SPC [10]. The important performance for control charts is average run length (ARL). The in-control process (ARL_0) - the expectation of the sample is taken to signal about a possible change, ARL_0 should be enough large. The out-of-control process (ARL_1) - the expectation of the sample is out-of control limit, ARL_1 should be a small. However, the previous research did not study the design methods

of DEWMA chart with transformed exponential observations and its performance compared to EWMA chart.

In this study is to propose the performance of DEWMA chart with transformed exponential observations and compare with DEWMA chart. The one-sided DEWMA chart with transformed exponential observations is investigated in section 2, and the performance of charts comparisons is presented in section 3. In section 4, conclusion is presented.

2. DEWMA Chart with Transformed Exponential Observations

We assume that the data $y_1, y_2, \dots, y_n, \dots$ are independent and identity exponential distribution with probability density function:

$$f(y) = \frac{1}{\alpha} e^{-\frac{y}{\alpha}}, \quad y \geq 0, \alpha > 0$$

where α is the mean. The exponential random variable y_n is transformed to Weibull random variable z_n ($z_n \sim W(\alpha^{0.25}, 4)$) using the double square root transformation which mean is $0.9064 \alpha^{0.25}$ and variance is $0.2543 \alpha^{0.25}$ [11]:

$$z_n = y_n^{\frac{1}{4}}, \quad y \geq 0.$$

The cumulative distribution function of z can be expressed as

$$F(z) = 1 - e^{-\left(\frac{y}{\alpha^{0.25}}\right)^4}$$

If $\alpha = \alpha_0$ and α_0 is known, then the process is in in-control and $\alpha = \alpha_1$ then the process is out-of-control. The DEWMA statistic D_n is defined with the recursion equations:

$$\begin{aligned} D_n &= (1 - \lambda) D_{n-1} + \lambda E_n, \quad n \geq 1, \\ E_n &= (1 - \lambda) E_{n-1} + \lambda Z_n, \quad n \geq 1. \end{aligned} \tag{1}$$

Where $0 < \lambda \leq 1$ is smoothing constant, the starting values D_0 and E_0 are initial values according to desired design conditions. Note that E_n is usual EWMA statistic and DEWMA statistic D_n is an EWMA of E_n . [4] and [7] shown the mean and variance of D_n ,

$$\begin{aligned} \mu_D &= \mu_E \\ \sigma_D^2 &= \mu_D \frac{\lambda^4 \left[1 + (1 - \lambda)^2 - (n + 1)^2 (1 - \lambda)^{2n} + (2n^2 + 2n - 1)(1 - \lambda)^{2n+2} - n^2 (1 - \lambda)^{2n+4} \right]}{\left[1 - (1 - \lambda)^2 \right]^3} \end{aligned}$$

The asymptotic variance of DEWMA as follow:

$$\sigma_D^2 = \sigma_E^2 \frac{\lambda(2 - 2\lambda + \lambda^2)}{(2 - \lambda)^3}.$$

The upper control limits (UCL) and lower control limit (LCL) can be calculated by

$$\begin{aligned} UCL &= \mu_D + L\sigma_D = H_D \\ LCL &= \mu_D - L\sigma_D = 0 \end{aligned}$$

where $L > 0$ is a control limit constant.

3. A Comparison the performance of charts

In this section, we compare the performance of one-sided DEWMA and EWMA charts with transformed exponential observations by using ARLs. For transformed exponential EWMA statistic is defined in Eq. (1) and the control limits are

$$\begin{aligned} UCL &= \mu_E + K\sigma_E = H_E \\ LCL &= \mu_E - K\sigma_E = 0 \end{aligned}$$

where $K > 0$ is a control limit constant.

We will consider smoothing constants $\lambda = 0.1, 0.2, 0.3, 0.4, 0.5$ for the one-sided DEWMA and the one-sided EWMA charts with transformed exponential observations and consider the exponential mean with $\alpha_0 = 1$ for the process is in control and when the process is out-of-control $\alpha_1 = 1.01, 1.03, 1.05, 1.07, 1.09, 1.1, 1.3, 1.5, 1.7, 1.9, 2, 2.5, 3, 5, 7, 9, 10$. While it is possible to find the values for the one-sided EWMA chart with transformed exponential observations using a Markov chain approach [11], the values for the DEWMA chart are more difficult to obtain using that approach [6]. For the purpose of DEWMA chart evaluation, we obtain the values through simulation and then obtained the ARLs and standard deviations of run lengths (SRLs). We report here results from 10,000 simulations per run for one-sided DEWMA chart. For one-sided EWMA chart, we obtained the ARLs.

Table 1. H_D values for DEWMA and H_E for EWMA charts when $\lambda = 0.1, 0.2, 0.3, 0.4$ and 0.5 ; $ARL_0 = 100$ and 300 . The in-control process mean α_0 is 1. The entries inside the parentheses are SRLs.

λ	H_D	ARL_0	λ	H_E	ARL_0
0.1	0.932	100.274 (57.738)	0.1	1.445	100
0.2	0.992	100.516 (79.645)	0.2	1.822	100
0.3	1.040	100.597 (88.729)	0.3	2.173	100
0.4	1.085	100.156 (91.017)	0.4	2.515	100
0.5	1.130	100.079 (93.370)	0.5	2.854	100
0.1	0.975	299.160 (229.811)	0.1	1.634	300
0.2	1.033	299.684 (250.596)	0.2	2.110	300
0.3	1.083	300.604 (259.956)	0.3	2.557	300
0.4	1.132	300.979 (263.307)	0.4	2.995	300
0.5	1.180	300.307 (266.519)	0.5	3.435	300

Table 2. H_D values for DEWMA and H_E for EWMA charts when $\lambda = 0.1, 0.2, 0.3, 0.4$ and 0.5 ; $ARL_0 = 500$. The in-control process mean α_0 is 1. The entries inside the parentheses are SRLs.

λ	H_D	ARL_0	λ	H_E	ARL_0
0.1	0.991	500.041 (325.222)	0.1	1.714	500
0.2	1.051	500.390 (340.196)	0.2	2.236	500
0.3	1.104	500.817 (343.831)	0.3	2.728	500
0.4	1.154	499.798 (345.940)	0.4	3.213	500
0.5	1.206	500.377 (346.124)	0.5	3.699	500

In Table 1 and 2, we show the (λ, H_D) and (λ, H_E) pairs that give an $ARL_0 = 100, 300$ and 500 of DEWMA and EWMA charts with transformed exponential observations, respectively. We can see that the control limits of EWMA are bigger than DEWMA in the same parameter λ .

Table 3. The ARL_1 of one-sided DEWMA and EWMA charts with transformed exponential observations. The entries inside the parentheses are SRLs.

α_1	DEWMA	EWMA	DEWMA	EWMA
	$\lambda = 0.1$	$\lambda = 0.1$	$\lambda = 0.3$	$\lambda = 0.3$
	$H_D = 0.991$	$H_E = 1.714$	$H_D = 1.104$	$H_E = 2.728$
1.01	471.355 (318.023)	486.080	476.322 (338.697)	489.197
1.03	393.725 (287.538)	460.532	421.147 (318.173)	470.038
1.05	345.788 (260.390)	437.043	374.865 (297.211)	452.093
1.07	291.780 (224.586)	415.399	322.507 (272.669)	435.259
1.09	251.323 (193.856)	395.415	284.082 (248.292)	419.446
1.1	238.184 (183.651)	385.994	258.454 (228.400)	411.897
1.3	93.101 (51.253)	254.012	89.332 (76.589)	298.402
1.5	61.181 (23.419)	183.413	46.781 (34.904)	229.743
1.7	48.922 (13.715)	141.198	30.770 (20.783)	184.748
1.9	42.458 (9.588)	113.836	23.417 (14.062)	153.466
2	40.236 (8.287)	103.600	20.955 (11.609)	141.239
2.5	33.519 (4.866)	71.183	14.586 (6.364)	100.212
3	29.993 (3.502)	54.396	11.886 (4.220)	77.305
5	23.792 (2.205)	29.552	8.072 (1.992)	40.916
7	21.087 (1.804)	21.580	6.778 (1.434)	28.710
9	19.449 (1.594)	17.638	6.094 (1.216)	22.683
10	18.839 (1.529)	16.322	5.825 (1.111)	20.686

In Table 3, we compare the ARL_1 both for one-sided DEWMA and EWMA charts with transformed exponential observations when ARL_0 is 500. We observe that DEWMA chart shows a better performance than EWMA chart with $\lambda = 0.3, H_D = 1.104$ and $H_E = 2.728$, . If $\lambda = 0.1, H_D = 0.991$ and

$H_k = 1.714$, DEWMA chart shows a better performance than EWMA chart, except for $\alpha_1 = 7$ the DEWMA chart performs similar to the EWMA chart in detecting large process mean increase.

4. Conclusion

In this paper, we have investigated DEWMA chart for monitoring transformed exponential observations. Using simulations, we have been able to provide table for boundaries and approximations ARLs for one-sided DEWMA chart with transformed exponential observations. The performance comparison of the charts has been based on ARL_1 criteria. For exponential observations when given $ARL_0 = 500$, we have shown that the performance of DEWMA chart is superior to EWMA chart for small change.

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